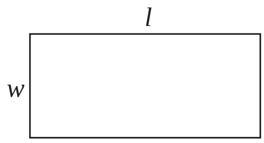
2 Formulae

2.1 Using Formulae

In formulae, letters are used to represent numbers. For example, the formula

$$A = lw$$

can be used to find the area of a rectangle. Here A is the area, l the length and w the width. In this formula, lw means $l \times w$. Formulae are usually written in this way without multiplication signs.



The perimeter of the rectangle would be given by the formula

$$P = 2l + 2w$$

Here again there are no multiplication signs, and $2l$ means $2 \times l$ and $2w$ means $2 \times w$.

Worked Example 1

The perimeter of a rectangle can be found using the formula

$$P = 2l + 2w$$

Find the perimeter if $l = 8$ and $w = 4$.

Solution

The letters l and w should be replaced by the numbers 8 and 4.

This gives

$$\begin{aligned} P &= 2 \times 8 + 2 \times 4 \\ &= 16 + 8 \\ &= 24 \end{aligned}$$

Worked Example 2

The final speed of a car is v and can be calculated using the formula

$$v = u + at$$

where u is the initial speed, a is the acceleration and t is the time taken.

Find v if the acceleration is 2 m s^{-1}, the time taken is 10 seconds and the initial speed is 4 m s^{-1}.

Solution

The acceleration is 2 m s^{-1} so $a = 2$. The initial speed is 4 m s^{-1} so $u = 4$.

The time taken is 10 s so $t = 10$.

Using the formula

$$v = u + at$$

gives

$$\begin{aligned} v &= 4 + 2 \times 10 \\ &= 4 + 20 \\ &= 24 \text{ m s}^{-1} \end{aligned}$$

Exercises

1. The area of a rectangle is found using the formula $A = lw$ and the perimeter using $P = 2l + 2w$. Find the area and perimeter if:

   (a) $l = 4$ and $w = 2$ (b) $l = 10$ and $w = 3$

   (c) $l = 11$ and $w = 2$ (d) $l = 5$ and $w = 4$

2. The formula $v = u + at$ is used to find the final speed.

   Find v if`:

   (a) $u = 6$, $a = 2$ and $t = 5$ (b) $u = 0$, $a = 4$ and $t = 3$

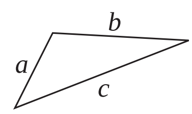
   (c) $u = 3$, $a = 1$ and $t = 12$ (d) $u = 12$, $a = 2$ and $t = 4$

3. Use the formula $F = ma$ to find F if:

   (a) $m = 10$ and $a = 3$ (b) $m = 200$ and $a = 2$

4. The perimeter of a triangle is found using the formula

   $$P = a + b + c$$

   Find P if:

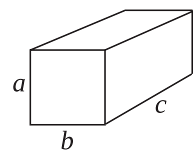
   (a) $a = 10$, $b = 12$ and $c = 8$

   (b) $a = 3$, $b = 4$ and $c = 5$

   (c) $a = 6$, $b = 4$ and $c = 7$

5. The volume of a box is given by the formula

   $$V = abc$$

   Find V if:

   (a) $a = 2$, $b = 3$ and $c = 10$

   (b) $a = 7$, $b = 5$ and $c = 3$

   (c) $a = 4$, $b = 4$ and $c = 9$

6. Find the value of Q for each formula using the values given.

   (a) $Q = 3x + 7y$
$x = 4$ and $y = 2$ (b) $Q = x^2 + y$
$x = 3$ and $y = 5$

   (c) $Q = xy + 4$
$x = 3$ and $y = 5$ (d) $Q = 5x - 2y$
$x = 10$ and $y = 2$

(e) $Q = xy - 2$
$x = 10$ and $y = 2$

(f) $Q = \dfrac{x}{y}$
$x = 24$ and $y = 2$

(g) $Q = \dfrac{x + 4}{y}$
$x = 8$ and $y = 3$

(h) $Q = \dfrac{4x + 2}{y}$
$x = 5$ and $y = 11$

(i) $Q = 3x + 2y + z$
$x = 4$, $y = 2$ and $z = 10$

(j) $Q = xy + yz$
$x = 2$, $y = 5$ and $z = 8$

(k) $Q = xyz$
$x = 2$, $y = 5$ and $z = 3$

(l) $Q = xy + 4z$
$x = 8$, $y = 3$ and $z = 4$

(m) $Q = \dfrac{x + y}{z}$
$x = 8$, $y = 10$ and $z = 3$

(n) $Q = \dfrac{x}{y + z}$
$x = 50$, $y = 2$ and $z = 3$

7. This formula is used to work out Sharon's pay.

Sharon works for 40 hours.
Her rate of pay is £3 per hour.

Pay = Number of hours worked $\times$ Rate of pay + £10.

Work out her pay.

(LON)

8. A rectangle has a length of *a* cm and a width of *b* cm.

The perimeter of a rectangle is given by the formula $p = 2(a + b)$.

Calculate the perimeter of a rectangle when $a = 4.5$ and $b = 4.2$.

(SEG)

2.2 Construct and Use Simple Formulae

A *formula* describes how one quantity relates to one or more other quantities. For example, a formula for the area of a rectangle describes how to find the area, given the length and width of the rectangle.

The perimeter of the rectangle would be given by the formula

$$P = 2l + 2w$$

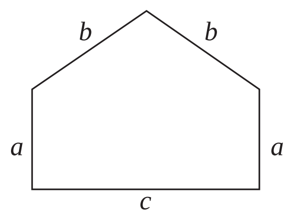
Here again there are no multiplication signs and $2l$ means $2 \times l$ and $2w$ means $2 \times w$.

Worked Example 1

(a) Write down a formula for the perimeter of the shape shown.

(b) Find the perimeter if

$a = 2$ cm, $b = 3$ cm and $c = 5$ cm



Solution

(a) The perimeter is found by adding together the lengths of all the sides, so the formula will be

$$P = a + b + b + a + c$$

but as a and b are both added in twice, this can be simplified to

$$P = 2a + 2b + c$$

(b) If $a = 2$, $b = 3$ and $c = 5$

$$\begin{aligned} P &= 2 \times 2 + 2 \times 3 + 5 \\ &= 4 + 6 + 5 \\ &= 15 \text{ cm} \end{aligned}$$

Worked Example 2

An emergency engineer charges a basic fee of £20, plus £8 per hour, when repairing central heating systems.

Find a formula for calculating the engineer's charge.

Solution

Let C = charge and n = number of hours.

The charge is made up of

a fixed £20 and £8 $\times$ the number of hours, or £$8n$.

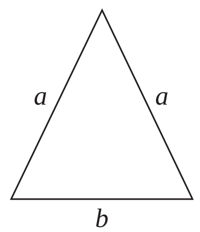
So the total charge is given by

$$C = 20 + 8n$$

Exercises

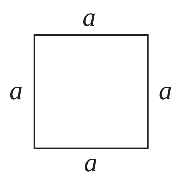
1. Find a formula for the perimeter of each shape, and find the perimeter for the specified values.

(a)



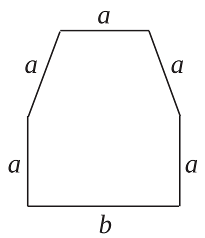
$a = 6$ cm, $b = 4$ cm

(b)



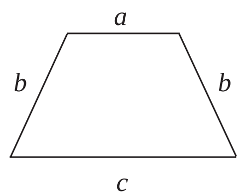
$a = 5$

(c)



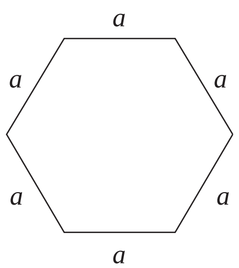
$a = 6$ cm, $b = 10$ cm

(d)



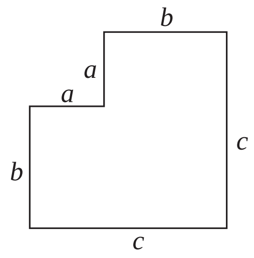
$a = 5$ cm, $b = 6$ cm, $c = 10$ cm

(e)



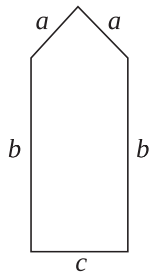
$a = 10$ cm

(f)



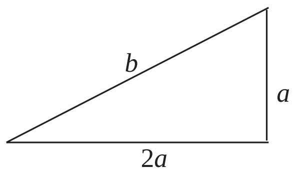
$a = 4$ cm, $b = 5$ cm, $c = 9$ cm

(g)



$a = 60$ cm, $b = 160$ cm,
$c = 80$ cm

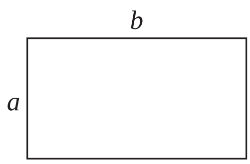
(h)



$a = 4$ cm, $b = 9$ cm

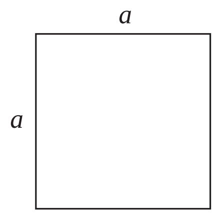
2. Find a formula for the area of each of the shapes below and find the area for the values given.

(a)



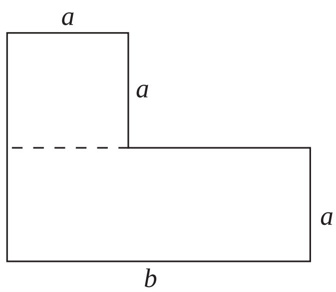
$a = 6$ cm, $b = 10$ cm

(b)



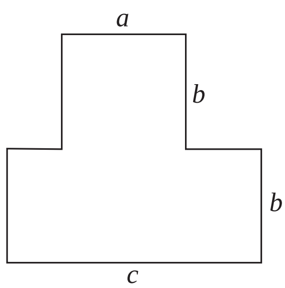
$a = 3$ cm

(c)



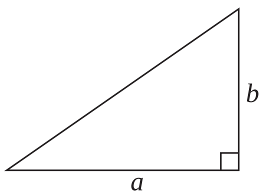
$a = 2$ cm, $b = 8$ cm

(d)



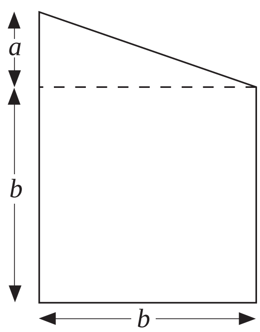
$a = 3$ cm, $b = 4$ cm, $c = 9$ cm

(e)



$a = 4$ cm, $b = 5$ cm

(f)



$a = 50$ cm, $b = 200$ cm

3. Three consecutive numbers are to be added together.

   (a) If x is the smallest number, what are the other two?

   (b) Write down a formula for the total, T, of the three numbers, using your answer to (a).

4. (a) Write down a formula to find the mean, M, of the two numbers x and y.

   (b) Write down a formula to find the mean, M, of the five numbers $p, q, r, s,$ and t.

5. Tickets for a school concert are sold at £3 for adults and £2 for children.

   (a) If p adults and q children buy tickets, write a formula for the total value, T, of the ticket sales.

   (b) Find the total value of the ticket sales if $p = 50$ and $q = 20$.

6. A rectangle is 3 cm longer than it is wide.

   If x is the width, write down a formula for:

   (a) the perimeter; P;

   (b) the area, A, of the rectangle.

7. Rachel is one year older than Ben. Emma is three years younger than Ben

   If Ben is x years old, write down expressions for:

   (a) Rachel's age;

   (b) Emma's age;

   (c) the sum of all three children's ages.

8. A window cleaner charges a fee of £3 for visiting a house and £2 for every window that he cleans.

   (a) Write down a formula for finding the total cost C when n windows are cleaned.

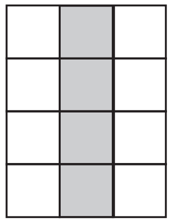
   (b) Find C if $n = 8$.

9. A taxi driver charges a fee of £1, plus £2 for every mile that the taxi travels.

   (a) Find a formula for the cost C of a journey that covers m miles.

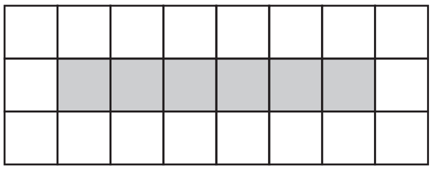
   (b) Find C if $m = 3$.

10. A gardener builds paths using paving slabs laid out in a pattern as shown, with white slabs on each side of a row of red slabs.



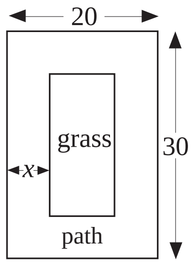
(a) If n red slabs are used, how many white slabs are needed?

(b) Another gardener puts a white slab at each end of the path as shown below.



If n red slabs are used, how many white slabs are needed?

11. A path of width x is laid around a rectangular lawn as shown.



(a) Find an expression for the perimeter of the grass.

(b) Find an expression for the area of the grass.

12. Choc Bars cost 27 pence each.
Write down a formula for the cost, C pence, of n Choc Bars.

(LON)

13. (a) Petrol costs 45 pence per litre.
Write down a formula for the cost, C pence, of l litres of petrol.

(b) Petrol costs x pence per litre.
Write down a formula for the cost, C pence, of l litres of petrol.

(SEG)

14. (a) Vijay earns £P in his first year of work.
The following year his salary is increased by £Q.

Write down an expression for his salary in his second year.

(b) Julie earns £X in her first year of work.
Her salary is increased by £650 every year.

How much will she earn in

(i) the 5th year (ii) the nth year?

2.3 Revision of Negative Numbers

Before starting the next section on formulae it is useful to revise how to work with negative numbers.

Note

When multiplying or dividing two numbers, if they have the *same* sign the result will be *positive*, but if they have *different* signs the result will be *negative*.

Worked Example 1

Find

(a) $(-3) \times (-7)$ (b) $(-24) \div 3$

(c) $(-40) \div (-5)$ (d) $(-6) \times 7$

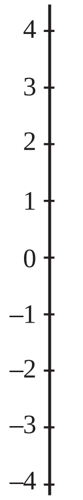
Solution

(a) $(-3) \times (-7) = 21$ (b) $(-24) \div 3 = -8$

(c) $(-40) \div (-5) = 8$ (d) $(-6) \times 7 = -42$

Note

When adding or subtracting it can be helpful to use a *number line*, remembering to move up when adding and down when subtracting a positive number. When adding a negative number, move down and when subtracting a negative number, move up.



Number line

Worked Example 2

Find

(a) $4 - 10$ (b) $-6 + 8$

(c) $-4 - 5$ (d) $-6 + (-7)$

(e) $7 - (-4)$

Solution

(a) $4 - 10 = -6$ (b) $-6 + 8 = +2$

(c) $-4 - 5 = -9$ (d) $-6 + (-7) = -6 - 7$
$= -13$

(e) $7 - (-4) = 7 + 4$
$= 11$

Exercises

1. (a) $6 - 8 =$ (b) $-8 + 12 =$ (c) $-5 + 2 =$

(d) $-6 - 2 =$ (e) $(-8) \times (-3) =$ (f) $(-9) \times (-6) =$

(g) $(-24) \div (-3) =$ (h) $16 \div (-2) =$ (i) $(-81) \div (-3) =$

(j) $-16 + 24 =$ (k) $-8 - 5 =$ (l) $(-5) \times 7 =$

(m) $3 \times (-8) =$ (n) $-1 - 10 =$ (o) $-10 + 5 =$

(p) $9 + (-6) =$ (q) $4 - (-7) =$ (r) $-1 - (-4) =$

(s) $-1 + (-7) =$ (t) $-4 + (-2) =$ (u) $-6 - (-5) =$

2. (a) $(-1)^2$ (b) $(-4)^2$ (c) $(-4)^2 + (-3)^2$

3. In Carberry, the temperature at midday was $5\ °C$.

   At midnight the temperature had fallen by $8\ °C$.

   What was the temperature at midnight?

   (MEG)

4. The temperature was recorded inside a house and outside a house.

Inside temperature	Outside temperature
16 °C	−8 °C

   How many degrees warmer was it inside the house than outside?

   (SEG)

2.4 Substitution into Formulae

The process of replacing the letters in a formula is known as *substitution*.

Worked Example 1

The length of a metal rod is l. The length changes with temperature and can be found by the formula

$$l = 40 + 0.02\,T$$

where T is the temperature.

Find the length of the rod when

(a) $T = 50\ °C$ and (b) $T = -10\ °C$

Solution

(a) Using $T = 50$ gives

$$\begin{aligned} l &= 40 + 50 \times 0.02 \\ &= 40 + 1 \\ &= 41 \end{aligned}$$

(b) Using $T = -10$ gives

$$\begin{aligned} l &= 40 + (-10) \times 0.02 \\ &= 40 + (-0.2) \\ &= 40 - 0.2 \\ &= 39.8 \end{aligned}$$

Worked Example 2

The profit made by a salesman when he makes sales on a day is calculated with the formula

$$P = 4n - 50$$

Find the profit if he makes

(a) 30 sales (b) 9 sales

Solution

(a) Here $n = 30$ so the formula gives

$$\begin{aligned} P &= 4 \times 30 - 50 \\ &= 120 - 50 \\ &= 70 \end{aligned}$$

(b) Here $n = 9$ so the formula gives

$$\begin{aligned} P &= 4 \times 9 - 50 \\ &= 36 - 50 \\ &= -14 \end{aligned}$$

So a loss is made if only 9 sales are made.

Exercises

1. The formula below is used to convert temperatures in degrees Celsius to degrees Fahrenheit, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

$$F = 1.8\,C + 32$$

Find F if:

(a) $C = 10$ (b) $C = 20$ (c) $C = -10$

(d) $C = -5$ (e) $C = -20$ (f) $C = 15$

2. The formula

$$s = \frac{1}{2}(u + v)t$$

is used to calculate the distance, s, that an object travels if it starts with a velocity u and has a velocity v, t seconds later.

Find s if:

(a) $u = 2,\ v = 8,\ t = 2$ (b) $u = 3,\ v = 5,\ t = 10$

(c) $u = 1.2,\ v = 3.8,\ t = 4.5$ (d) $u = -4,\ v = 8,\ t = 2$

(e) $u = 4,\ v = -8,\ t = 5$ (f) $u = 1.6,\ v = 2.8,\ t = 3.2$

3. The length, l, of a spring is given by the formula

$$l = 20 - 0.08F$$

where F is the size of the force applied to the spring to compress it.

Find l if:

(a) $F = 5$ (b) $F = 20$

(c) $F = 24$ (d) $F = 15$

4. The formula

$$P = 120n - 400$$

gives the profit, P, made when n cars are sold in a day at a showroom.

Find P if:

(a) $n = 1$ (b) $n = 3$

(c) $n = 4$ (d) $n = 10$

How many cars must be sold to make a profit?

5. Work out the value of each function by substituting the values given, *without* using a calculator.

(a) $V = p^2 + q^2$
$p = 8$ and $q = 4$

(b) $p = a^2 - b^2$
$a = 10$ and $b = 7$

(c) $z = \sqrt{x + y}$
$x = 10$ and $y = 6$

(d) $Q = \sqrt{x - y}$
$x = 15$ and $y = 6$

(e) $P = \dfrac{x + y}{2}$
$x = 4$ and $y = -10$

(f) $Q = \sqrt{\dfrac{a}{b}}$
$a = 100$ and $b = 4$

(g) $V = \dfrac{x + 2y + z}{5}$
$x = 2,\ y = -5$ and $z = 8$

(h) $R = \dfrac{1}{a} + \dfrac{1}{b}$
$a = 4$ and $b = 2$

(i) $S = \dfrac{a}{b} + \dfrac{b}{c}$
$a = 3,\ b = 4$ and $c = 16$

(j) $R = 0.2a + 0.4b$
$a = 10$ and $b = 20$

(k) $T = \dfrac{a}{2} + \dfrac{b}{5}$
$a = -20$ and $b = 40$

(l) $C = \dfrac{ab}{a + b}$
$a = 10$ and $b = -5$

(m) $P = 2\sqrt{\dfrac{x^2}{y}}$
$x = 10$ and $y = 4$

(n) $A = \dfrac{ab^2}{c}$
$a = 2,\ b = 3$ and $c = 100$

(o) $X = \dfrac{b + c}{a}$

$a = 10$, $b = 1.7$ and $c = 2.1$

(p) $z = \sqrt{x^2 + y^2}$

$x = -3$ and $y = 4$

(q) $P = \sqrt{a^2 - b^2}$

$a = -10$ and $b = 6$

(r) $Q = \sqrt{x^2 + y^2 + z^2}$

$x = -10$, $y = 5$ and $z = 10$

6. Work out the value of each function by substituting the values given, using a calculator if necessary.

(a) $P = \dfrac{x - y}{z}$

$x = 10$, $y = 2.02$ and $z = 2.1$

(b) $V = \dfrac{x - y}{x + y}$

$x = 4.9$ and $y = 3.1$

(c) $R = \dfrac{x^2 - y^2}{4}$

$x = 3.6$ and $y = 1.6$

(d) $D = \dfrac{2}{x} + \dfrac{2}{y}$

$x = 0.4$ and $y = 0.8$

(e) $Q = \dfrac{x^2 + y^2}{5}$

$x = 3.7$ and $y = 5.9$

(f) $V = \dfrac{3x + 2y}{x + y}$

$x = 1.6$ and $y = 2.4$

(g) $R = \dfrac{p + q}{p - q}$

$p = 1.2$ and $q = -0.4$

(h) $A = \dfrac{x^2 + y^2}{x + y}$

$x = 5.2$ and $y = -1.2$

(i) $P = \sqrt{x - \dfrac{y}{10}}$

$x = 3.09$ and $y = -106$

7. The formula to convert temperatures from degrees Fahrenheit ($°F$) into degrees Celsius ($°C$) is

$$C = \frac{5}{9}(F - 32)$$

Calculate the temperature in degrees Celsius which is equivalent to a temperature of $-7\,°F$.

(MEG)

8. $$F = \frac{9R}{4} + 32$$

Calculate the value of F when $R = -20$.

(LON)

9. Given that $m = \frac{1}{2}$, $p = \frac{3}{4}$, $t = -2$, calculate

(a) $m\,p + t$ (b) $\frac{(m + p)}{t}$

(NEAB)

2.5 More Complex Formulae

Some formulae such as

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \text{and} \quad z^2 = x^2 + y^2$$

arise in science or mathematics, but when used do not lead directly to values of f or z.

Worked Example 1

Use the formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

to find f if $u = 10$ and $v = 8$.

Solution

Substituting into the formula gives

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{8}$$

First add together the two fractions using 40 as a common denominator:

$$\frac{1}{f} = \frac{4}{40} + \frac{5}{40}$$

$$\frac{1}{f} = \frac{9}{40}$$

Now to find f, turn both fractions upside-down to give

$$\frac{f}{1} = \frac{40}{9} \qquad \text{or} \qquad f = 4\frac{4}{9}$$

Worked Example 2

Find z using the formula

$$z^2 = x^2 + y^2$$

if $x = 3.6$ and $y = 4.8$.

Solution

Substituting these values into the formula gives

$$z^2 = 3.6^2 + 4.8^2$$

$$z^2 = 12.96 + 23.04$$

$$z^2 = 36$$

Now the square root can be taken of both sides to give

$$z = +\sqrt{36} \text{ or } -\sqrt{36}$$

$$z = 6 \text{ or } -6$$

Exercises

1. Use the formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

to find f if:

(a) $v = 3$ and $u = 4$

(b) $v = 6$ and $u = -5$

(c) $v = 7$ and $u = -3$

(d) $v = 10$ and $u = -4$

2. Find z using the formula

$$z^2 = x^2 + y^2$$

if:

(a) $x = 1.2$ and $y = 0.5$

(b) $x = 4.8$ and $y = 6.4$

(c) $x = 3$ and $y = 1.6$

3. Find the value of z as a fraction or mixed number in each case below.

(a) $\frac{1}{z} = \frac{x}{x+y}$
$x = 4$ and $y = -10$

(b) $\frac{1}{z} = \frac{x}{y} + \frac{y}{x}$
$x = 3$ and $y = 4$

(c) $\frac{1}{z} = \frac{2}{x} + \frac{3}{y}$
$x = 4$ and $y = -5$

(d) $\frac{1}{z} = \frac{x-y}{x+y}$
$x = -7$ and $y = -3$

(e) $\frac{1}{z} = \frac{x}{4} + \frac{3}{y}$
$x = 5$ and $y = -2$

(f) $\frac{1}{z} = \frac{1+x}{1-x}$
$x = 2$

(g) $\frac{1}{z} = \frac{x-2}{x+4}$

$x = \frac{1}{4}$

(h) $\frac{x+y}{x-y} = \frac{1}{z}$

$x = 4$ and $y = \frac{1}{2}$

(i) $\frac{x}{2} + \frac{3}{y} = \frac{1}{z}$

$x = 1$ and $y = 6$

4. Find z in each case below.

(a) $z^2 = 9 + x^2$

$x = 4$

(b) $z^2 = x + y$

$x = 147$ and $y = -3$

(c) $z^2 = x - y$

$x = 44$ and $y = -5$

(d) $z^2 = \frac{x}{y}$

$x = 363$ and $y = 3$

(e) $z^2 = \frac{x+6}{y}$

$x = 6$ and $y = 3$

(f) $z^2 = \frac{x}{8+y}$

$x = 16.9$ and $y = -7.9$

5. When three resistors are connected in parallel the total resistance R is given by

$$\frac{1}{R} = \frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$$

where X, Y and Z are the resistances of each resistor.

Find R if:

(a) $X = 10$, $Y = 20$ and $Z = 30$

(b) $X = 1000$, $Y = 5000$ and $Z = 2000$

(c) $X = 1500$, $Y = 2200$ and $Z = 1600$

6. Use the formula

$$y = \frac{x-1}{\sqrt{\left(t - v^2\right)}}$$

to calculate the value of y given that

$x = 50$, $t = 2.5$ and $v = 0.6$

Give your answer correct to 1 decimal place.

Show all necessary working.

(LON)

7. The formula $f = \dfrac{uv}{u+v}$ is used in the study of light.

(a) Calculate f when $u = 14.9$ and $v = -10.2$.
Give your answer correct to 3 significant figures.

(b) By rounding the values of u and v in part (a) to 2 significant figures, check whether your answer to part (a) is reasonable.
Show your working.

(MEG)

8. A ball bearing has mass 0.44 pounds.

$$1 \text{ kg} = 2.2 \text{ pounds}.$$

(a) (i) Calculate the mass of the ball bearing in kilograms.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

(ii) When the mass of the ball bearing is measured in kg and the volume is measured in cm^3, what are the units of the density?

(b) The volume of a container is given by the formula

$$V = 4L(3-L)^2$$

Using Mass = Volume × Density calculate the mass of the container when $L = 1.40$ cm, and 1 cm^3 of the material has a mass of 0.160 kg.

2.6 Changing the Subject

Sometimes a formula can be rearranged into a more useful format. For example, the formula

$$F = 1.8C + 32$$

can be used to convert temperatures in degrees Celsius to degrees Fahrenheit. It can be rearranged into the form

$$C = \ldots$$

to enable temperatures in degrees Fahrenheit to be converted to degrees Celsius. We say that the formula has been rearranged to make C the subject of the formula.

Worked Example 1

Rearrange the formula

$$F = 1.8C + 32$$

to make C the subject of the formula.

Solution

The aim is to remove all terms from the right hand side of the equation except for the C. First subtract 32 from both sides, which gives

$$F - 32 = 1.8C$$

Then dividing both sides by 1.8 gives

$$\frac{F - 32}{1.8} = C$$

So the formula can be rearranged as

$$C = \frac{F - 32}{1.8}$$

Worked Example 2

Make v the subject of the formula

$$s = \frac{(u + v)t}{2}$$

Solution

First multiply both sides of the formula by 2 to give

$$2s = (u + v)t$$

Then divide both sides by t, to give

$$\frac{2s}{t} = u + v$$

Finally, subtract u from both sides to give

$$\frac{2s}{t} - u = v$$

So the formula becomes

$$v = \frac{2s}{t} - u$$

Exercises

1. Make x the subject of each of the following formulae.

(a)	$y = 4x$	(b)	$y = 2x + 3$	(c)	$y = 4x - 8$
(d)	$y = \frac{x + 2}{4}$	(e)	$y = \frac{x - 2}{5}$	(f)	$y = x + a$
(g)	$y = \frac{x - b}{a}$	(h)	$y = ax + c$	(i)	$y = \frac{ax + b}{c}$
(j)	$y = \frac{ax - c}{b}$	(k)	$y = a + b + x$	(l)	$y = \frac{x - a + b}{c}$

(m) $y = abx$ (n) $y = abx + c$ (o) $y = \dfrac{4ax - b}{3c}$

(p) $p = \dfrac{ax - bc}{d}$ (q) $y = (a + x)b$ (r) $y = \dfrac{(3 + x)a}{4}$

(s) $q = \dfrac{3(x - 4)}{2}$ (t) $v = \dfrac{5(x + y)}{4}$ (u) $z = a + \dfrac{(x - 3)}{4}$

2. Ohm's law is used in electrical circuits and states that

$$V = IR$$

Write formulae with *I* and *R* as their subjects.

3. Newton's Second law states that $F = ma$.

Write formulae with *m* and *a* as their subjects.

4. The formula $C = 2\pi r$ can be used to find the circumference of a circle. Make *r* the subject of this formula.

5. The equation $v = u + at$ is used to find the velocities of objects.

(a) Make *t* the subject of this formula.

(b) Make *a* the subject of this formula.

6. The mean of three numbers *x*, *y* and *z* can be found using the formula

$$m = \frac{x + y + z}{3}$$

Make *z* the subject of this formula.

7. Make *a* the subject of the following formulae.

(a) $v^2 = u^2 + 2as$ (b) $s = at + \dfrac{1}{2}at^2$

8. The formula $V = xyz$ can be used to find the volume of a rectangular box. Make *z* the subject of this formula.

9. The volume of a tin can is given by

$$V = \pi r^2 h$$

where *r* is the radius of the base and *h* is the height of the can.

(a) Make *r* the subject of the equation.

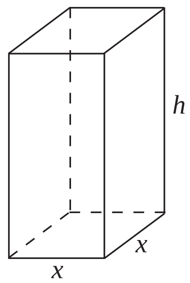
(b) Find *r* correct to 2 decimal places if $V = 250\ \text{cm}^3$ and $h = 10\ \text{cm}$.

10. A box with a square base has its volume given by

$$V = x^2 h$$

and its surface area given by

$$A = 2x^2 + 4xh$$



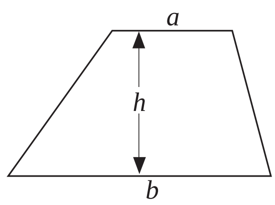
(a) Make h the subject of both formulae.

(b) Find h if $A = 24\text{ cm}^2$ and $x = 2\text{ cm}$.

(c) Find h if $V = 250\text{ cm}^3$ and $x = 10\text{ cm}$.

11. The area of a trapezium is given by

$$A = \frac{1}{2}(a + b)h$$



(a) Write the formula with a as its subject.

(b) In a particular trapezium $b = 2a$.
Use this to write a formula that does not involve b, and make a the subject.

2.7 Further Change of Subject

This section uses some further approaches to rearranging formulae.

Worked Example 1

Make l the subject of the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solution

First divide both sides by 2π to give

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

Now the square root can be easily removed by squaring both sides of the equation, to give

$$\frac{T^2}{4\pi^2} = \frac{l}{g}$$

Finally, both sides can be multiplied by g to give

$$\frac{T^2 g}{4\pi^2} = l$$

so the rearranged formula is

$$l = \frac{T^2 g}{4\pi^2}$$

Worked Example 2

Make x the subject of the formula

$$y = 6 - 5x$$

Solution

To avoid leaving $-5x$ on the right hand side of the formula, first add $5x$ to both sides to give

$$y + 5x = 6$$

Then subtract y from both sides to give

$$5x = 6 - y$$

Finally, divide by 5 to give

$$x = \frac{6 - y}{5}$$

Worked Example 3

Make x the subject of the formula

$$q = \frac{1}{x} + \frac{1}{y}$$

Solution

First subtract $\frac{1}{y}$ from both sides so that the right hand side contains only terms involving x.

$$q - \frac{1}{y} = \frac{1}{x}$$

Now combine the two terms on the left hand side of the formula into a single fraction, by first making y the common denominator.

$$\frac{qy}{y} - \frac{1}{y} = \frac{1}{x}$$

$$\frac{qy - 1}{y} = \frac{1}{x}$$

Now both fractions can be turned upside-down to give

$$\frac{x}{1} = \frac{y}{qy - 1}$$

or

$$x = \frac{y}{qy - 1}$$

Exercises

1. Rearrange each of the following formulae so that x is the subject.

(a) $y = 5 - 3x$ (b) $y = 8 - 6x$ (c) $y = a - 2x$

(d) $y = \dfrac{6 - 2x}{5}$ (e) $y = \dfrac{8 - 7x}{2}$ (f) $y = \dfrac{7x - 5}{3}$

(g) $p = \dfrac{a - x - b}{2}$ (h) $q = \dfrac{8 - x + 2}{a}$ (i) $r = \dfrac{q - 5x}{b}$

2. For each formula below make a the subject.

(a) $q = \sqrt{\dfrac{a}{4}}$ (b) $z = \sqrt{\dfrac{a}{b}}$ (c) $z = \sqrt{\dfrac{c}{a}}$

(d) $y = 2\sqrt{\dfrac{2a}{3}}$ (e) $v = \dfrac{1}{4}\sqrt{\dfrac{a}{2b}}$ (f) $r = 5\sqrt{\dfrac{a}{\pi}}$

(g) $p = \sqrt{\dfrac{a + b}{4}}$ (h) $r = \dfrac{1}{2}\sqrt{\dfrac{b - a}{3}}$ (i) $c = 3\sqrt{\dfrac{2}{b + a}}$

3. Make u the subject of each of the following formulae.

(a) $a = \dfrac{1}{u} + \dfrac{1}{2}$ (b) $b = \dfrac{1}{u} - 2$ (c) $x = 2 - \dfrac{1}{u}$

(d) $\dfrac{1}{x} = \dfrac{1}{u} + \dfrac{1}{3}$ (e) $\dfrac{1}{p} = \dfrac{1}{u} - \dfrac{1}{5}$ (f) $\dfrac{1}{x} = \dfrac{2}{u} + \dfrac{1}{3}$

(g) $\dfrac{1}{r} = \dfrac{4}{u} + \dfrac{2}{v}$ (h) $\dfrac{1}{q} = \dfrac{1}{7} - \dfrac{1}{u}$ (i) $\dfrac{1}{p} = \dfrac{1}{a} - \dfrac{1}{u}$

4. The formula $T = 2\pi\sqrt{\dfrac{l}{g}}$ gives the time for a pendulum to complete one full swing.

(a) Make g the subject of the formula.

(b) Find g if $l = 0.5$ and $T = 1.4$.

5. The formula $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ is used to find the focal length of a lens.

(a) Make v the subject of the formula.

(b) Find v if $f = 12$ and $u = 8$.

6. If a ball is dropped from a height, h, it hits the ground with speed, v, given by

$$v = \sqrt{2gh}$$

(a) Make h the subject of this formula.

(b) Find h if $g = 10$ and $v = 6$.

(c) Make g the subject of the formula.

(d) Find the value of g on a planet when $h = 10$, $v = 4$.

7. A ball is thrown so that it initially travels at $45°$ to the horizontal. If it travels a distance R, then its initial speed, u, is given by

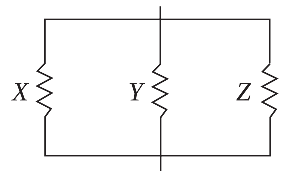
$$u = \sqrt{gR}$$

(a) Make R the subject of the formula.

(b) Find R if $u = 12$ and $g = 10$.

8. When three resistors with resistances X, Y and Z are connected as shown in the diagram, the total resistance is R, and

$$\frac{1}{R} = \frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$$



(a) Make X the subject of this equation.

(b) Find X if $R = 10$, $Y = 30$ and $Z = 40$.

9. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

(a) Rearrange the formula to give r, in terms of V.

(b) Find the value of r when $V = 75$.

(SEG)

2.8 Expansion of Brackets

An equation or formula may involve brackets as, for example, in $s = \frac{t}{2}(u + v)$.

Removing the brackets from such an expression is a process known as *expanding*.

Worked Example 1

Expand $4(x + 8)$.

Solution

Each term inside the bracket must be multiplied by the 4 that is in front of the bracket.

$$\begin{aligned} 4(x + 8) &= 4 \times x + 4 \times 8 \\ &= 4x + 32 \end{aligned}$$

Worked Example 2

Expand $x(x-2)$.

Solution

Every item in the bracket must be multiplied by x. This gives

$$\begin{aligned} x(x-2) &= x \times x + x \times (-2) \\ &= x^2 - 2x \end{aligned}$$

Worked Example 3

Expand $-2(4-3x)$.

Solution

Each term inside the bracket is this time multiplied by -2.

$$\begin{aligned} -2(4-3x) &= -2 \times 4 + (-2) \times (-3x) \\ &= -8 + 6x \end{aligned}$$

Worked Example 4

Expand and simplify $x(x-2) + 4(2x+1)$.

Solution

Each bracket must be expanded first and then like terms collected.

$$\begin{aligned} x(x-2) + 4(2x+1) &= x \times x - x \times 2 + 4 \times 2x + 4 \times 1 \\ &= x^2 - 2x + 8x + 4 \\ &= x^2 + 6x + 4 \end{aligned}$$

Exercises

1. Expand the following:

(a) $3(x+1)$	(b) $4(a+2)$	(c) $3(x-6)$
(d) $5(3-b)$	(e) $2(8-x)$	(f) $3(x+4)$
(g) $2(5x-12)$	(h) $6(2x-5)$	(i) $3(2x+7)$

2. Expand the following:

(a) $-2(x+6)$	(b) $-3(x+2)$	(c) $-6(x-3)$
(d) $-7(x-2)$	(e) $-4(2x+1)$	(f) $-5(3-2x)$
(g) $-2(3x-8)$	(h) $-3(-4-x)$	(i) $-8(2-4x)$

3. Expand the following:

(a) $x(x+1)$ (b) $x(1-x)$ (c) $x(x-6)$

(d) $-x(3x-2)$ (e) $-x(4x-6)$ (f) $a(4a+5)$

(g) $3a(2a-5)$ (h) $3y(4y-21)$ (i) $6y(5-2y)$

4. Expand and simplify the following:

(a) $3+2(x-8)$ (b) $x(x+1)-3x$

(c) $5(x+7)-12$ (d) $4(x+2)+2(x-1)$

(e) $3(x-6)+2(4x-5)$ (f) $4(n-2)+n(n+6)$

(g) $4(a+6)-2(a-2)$ (h) $3x(x-2)-4(x-6)$

(i) $2x(x+1)-x(7-x)$

5. Remove the brackets from each expression and simplify if possible.

(a) $x^2(x+1)$ (b) $2x(x^2-5x)$

(c) $\frac{1}{2}(4x+12)$ (d) $\frac{2}{3}(12x-6)$

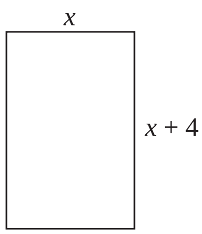
(e) $3x(2x^2-4)$ (f) $x(x^2+4)+x^2(3x+2)$

(g) $a(p+q)+p(a+b)$ (h) $3n(x+y)+4x(y-2n)$

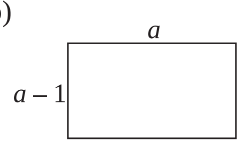
(i) $x(p+q)+p(x+q)-q(p-x)$

6. Find the area of each rectangle below.

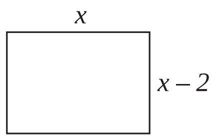
(a)



(b)



(c)



7. In a game, Stuart asks his friend to think of a number, add 1 to it and then double the result.

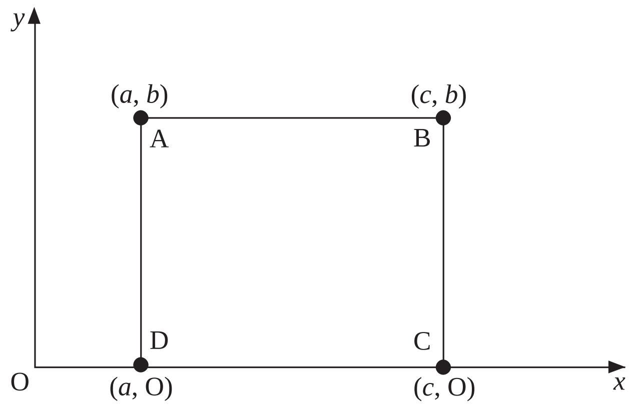
(a) using x to represent the unknown number, write Stuart's instructions, using brackets.

(b) Expand your answer to (a).

(c) Describe an alternative set of instructions that Stuart could use.

(d) Repeat (a) to (c) for an alternative game where Stuart asks his friend to think of a number, add 1 to it and then multiply the result by the number first thought of.

8.



(a) For the rectangle shown in the diagram, find the length BC.

(b) Find the length AB.

(c) Write an expression involving brackets for the area of the rectangle.

(d) Expand your answer to (c) to give an alternative expression.

2.9 Factorisation

The process of removing brackets is known as *expanding*. The reverse process of inserting brackets is known as *factorising*. To factorise an expression it is necessary to identify numbers or variables that are factors of all the terms.

Worked Example 1

Factorise

$$6x + 8$$

Solution

Both terms can be divided by 2, so it is factorised as:

$$\begin{aligned} 6x + 8 &= 2 \times 3x + 2 \times 4 \\ &= 2(3x + 4) \end{aligned}$$

Worked Example 2

Factorise

$$12a - 16$$

Solution

Here the largest number that both terms can be divided by is 4.

$$\begin{aligned} 12a - 16 &= 4 \times 3a - 4 \times 4 \\ &= 4(3a - 4) \end{aligned}$$

Worked Example 3

Factorise

$$4x^2 - 8x$$

Solution

Here 4 is the largest number that will divide into both terms but each term can also be divided by x, so $4x$ should be placed in front of the bracket.

$$\begin{aligned} 4x^2 - 8x &= 4x \times x - 4x \times 2 \\ &= 4x(x-2) \end{aligned}$$

Exercises

1. Complete a copy of each of the following.

(a) $5x + 10 = \ ?\ (x + 2)$ (b) $6x - 8 = \ ?\ (3x - 4)$

(c) $15x + 25 = \ ?\ (3x + 5)$ (d) $12x + 8 = 4(\ ? + ?\ )$

(e) $18 - 6n = 6(\ ? - ?\ )$ (f) $6x - 21 = 3(\ ? - ?\ )$

(g) $16a + 24 = 8(\ ? + ?\ )$ (h) $33x - 9 = 3(\ ? - ?\ )$

2. Factorise each of the following expressions.

(a) $6x + 24$ (b) $5x - 20$ (c) $16 - 8x$

(d) $8n + 12$ (e) $12x - 14$ (f) $3a - 24$

(g) $11x - 66$ (h) $10 + 25x$ (i) $100x - 40$

(j) $50 - 40x$ (k) $6x - 30$ (l) $5y - 45$

(m) $12 + 36x$ (n) $16x + 32$ (o) $27x - 33$

3. Complete a copy of each of the following.

(a) $x^2 + x = \ ?\ (x + 1)$ (b) $x^2 + 2x = \ ?\ (x + 2)$

(c) $2a^2 - 5a = \ ?\ (2a - 5)$ (d) $4x^2 + x = x(\ ? + ?\ )$

(e) $x^2 + 4x = x(\ ? + ?\ )$ (f) $6x^2 + 3x = 3x(\ ? + ?\ )$

(g) $xa + xb = x(\ ? + ?\ )$ (h) $4x^2 - 2ax = 2x(\ ? - ?\ )$

4. Factorise each of the following expressions.

(a)	$5x^2 + x$	(b)	$a^2 + 3a$	(c)	$5n^2 + 2n$
(d)	$6n^2 + 3n$	(e)	$5n^2 - 10n$	(f)	$3x^2 + 6x$
(g)	$15x^2 - 30x$	(h)	$14x^2 + 21x$	(i)	$16x^2 + 24x$
(j)	$30x^2 - 18x$	(k)	$5 + 5n^2$	(l)	$10n^2 - 15$
(m)	$3n^3 + 9n$	(n)	$9x^2 - 27x$	(o)	$10x^3 - 5x^2$

5. Factorise each of the following expressions.

(a)	$ax + ax^2$	(b)	$bx + cx^2$	(c)	$2pq - 4rq$
(d)	$15xy - 5y^2$	(e)	$16pq + 24p^2$	(f)	$6x^2 + 18xy$
(g)	$3p^2 - 9px$	(h)	$24px + 56x^2$	(i)	$16x^2y - 18xy^2$

6. For each factorisation shown below, state if it can be factorised further. If the answer is yes, give the complete factorisation.

(a)	$6x^2 + 4x = 2(3x^2 + 2x)$	(b)	$16x^3 + 8x^2 = 8x(2x^2 + x)$
(c)	$5x^2 - 60x = 5x(x - 12)$	(d)	$3x^2y - 18xy^2 = 3x(xy - 6y^2)$

7. (a) Factorise completely $36x + 6x^2$.

(b) Given that $y = 3 + 5x$

express x in terms of y.

2.10 Algebraic Manipulation

Sometimes a letter may appear twice in a formula, for example,

$$p = \sqrt{\frac{w}{x + w}}$$

This section is concerned with how to make the repeated letter the subject of the equation.

Worked Example 1

Make x the subject of the formula

$$ax - c = 3x + b$$

Solution

First bring all the terms containing x to one side of the equation. Subtracting $3x$ gives

$$ax - 3x - c = b$$

Then adding c to both sides gives

$$ax - 3x = b + c$$

Factorising gives

$$x(a-3)=b+c$$

Finally, dividing by $(a-3)$ gives

$$x=\frac{b+c}{a-3}$$

Worked Example 2

Make w the subject of the formula

$$p=\frac{2w}{x-w}$$

Solution

First multiply both sides by $x-w$ and expand the brackets.

$$p=\frac{2w}{x-w}$$

$$p(x-w)=2w$$

$$px-pw=2w$$

Next take all the terms containing w to one side of the equation and factorise.

$$px=2w+pw$$

$$px=w(2+p)$$

Finally, dividing by $(2+p)$ gives

$$\frac{px}{2+p}=w$$

or

$$w=\frac{px}{2+p}$$

Exercises

1. Make x the subject of each of the following formulae.

(a) $2x+a=x-b$

(b) $ax-b=cx-d$

(c) $xa-4=bx-5$

(d) $3x-6=4a+2x$

(e) $b-2x=c-5x$

(f) $a-bx=c-dx$

(g) $2(x+1)=a-x$

(h) $4(x-a)=3(a-x)$

(i) $p(x+1)=q(x-1)$

(j) $\dfrac{x-a}{2}=\dfrac{x+b}{3}$

(k) $\dfrac{2x-a}{5}=x+1$

(l) $\dfrac{x}{a}=\dfrac{x+b}{4}$

2. Make x the subject of each of the following formulae.

(a) $P = \dfrac{x}{x+1}$ (b) $P = \dfrac{ax+b}{x}$ (c) $Q = \dfrac{x+b}{x-a}$

(d) $q^2 = \dfrac{x+y}{x-y}$ (e) $\dfrac{x-2}{x+3} = a$ (f) $\dfrac{x-b}{x-c} = 4$

(g) $p = \sqrt{\dfrac{x}{x+1}}$ (h) $w = \sqrt{\dfrac{x-2}{x}}$ (i) $w = \sqrt{\dfrac{x-2}{x+1}}$

(j) $p = \dfrac{x^2+2}{x^2}$ (k) $p = \dfrac{2-x^2}{3-x^2}$ (l) $g = \dfrac{x^2-y}{x^2+y}$

2.11 Algebraic Fractions

When fractions are added or subtracted, a common denominator must be used as shown below:

$$\begin{aligned}\frac{1}{2}+\frac{1}{3} &= \frac{3}{6}+\frac{2}{6}\\ &= \frac{5}{6}\end{aligned}$$

When working with algebraic fractions a similar approach must be used.

Worked Example 1

Express

$$\frac{x}{6}+\frac{x}{5}$$

as a single fraction.

Solution

These fractions should be added by using a common denominator of 30.

$$\begin{aligned}\frac{x}{6}+\frac{x}{5} &= \frac{5x}{30}+\frac{6x}{30}\\ &= \frac{11x}{30}\end{aligned}$$

Worked Example 2

Express

$$\frac{3}{x}+\frac{4}{x+1}$$

as a single fraction.

Solution

In this case, the common denominator will be $x(x+1)$.
Using this gives

$$\begin{aligned}\frac{3}{x}+\frac{4}{x+1} &= \frac{3(x+1)}{x(x+1)}+\frac{4x}{x(x+1)} \\ &= \frac{3x+1}{x(x+1)}+\frac{4x}{x(x+1)} \\ &= \frac{7x+1}{x(x+1)}\end{aligned}$$

Worked Example 3

Express

$$\frac{3x}{2x+1}+\frac{x}{x+1}$$

as a single fraction.

Solution

In this case, the common denominator will be $(2x+1)(x+1)$.
Using this gives

$$\begin{aligned}\frac{3x}{2x+1}+\frac{x}{x+1} &= \frac{3x(x+1)}{(2x+1)(x+1)}+\frac{x(2x+1)}{(2x+1)(x+1)} \\ &= \frac{3x^2+3x}{(2x+1)(x+1)}+\frac{2x^2+x}{(2x+1)(x+1)} \\ &= \frac{5x^2+4x}{(2x+1)(x+1)}\end{aligned}$$

Exercises

1. Simplify each expression into a single fraction.

(a) $\frac{x}{4}+\frac{x}{5}=\ ?$ (b) $\frac{x}{7}+\frac{x}{4}=\ ?$ (c) $\frac{x}{3}+\frac{x}{5}=\ ?$

(d) $\frac{2y}{7}+\frac{5y}{3}=\ ?$ (e) $\frac{2y}{5}+\frac{3y}{4}=\ ?$ (f) $\frac{5y}{7}+\frac{8y}{7}=\ ?$

(g) $\frac{4x}{7}-\frac{3x}{10}=\ ?$ (h) $\frac{5x}{6}-\frac{2x}{3}=\ ?$ (i) $\frac{x}{4}+\frac{7x}{8}=\ ?$

(j) $\dfrac{5x}{6} + \dfrac{7x}{24} = ?$ (k) $\dfrac{a}{4} + \dfrac{b}{5} = ?$ (l) $\dfrac{x}{3} + \dfrac{y}{8} = ?$

(m) $\dfrac{a}{3} - \dfrac{b}{5} = ?$ (n) $\dfrac{2a}{3} + \dfrac{4b}{5} = ?$ (o) $\dfrac{8a}{9} - \dfrac{3b}{4} = ?$

2. Express the following as single fractions.

(a) $\dfrac{4}{x} + \dfrac{2}{y} = ?$ (b) $\dfrac{6}{x} - \dfrac{1}{y} = ?$ (c) $\dfrac{1}{x} + \dfrac{3}{y} = ?$

(d) $\dfrac{8}{a} - \dfrac{3}{a} = ?$ (e) $\dfrac{4}{a} + \dfrac{3}{2b} = ?$ (f) $\dfrac{5}{3a} - \dfrac{1}{2b} = ?$

(g) $\dfrac{5}{3a} + \dfrac{4}{5b} = ?$ (h) $\dfrac{7}{3a} - \dfrac{4}{5a} = ?$ (i) $\dfrac{6}{7a} - \dfrac{1}{4a} = ?$

(j) $\dfrac{7}{8a} - \dfrac{2}{3b} = ?$ (k) $\dfrac{3}{6a} - \dfrac{5}{12a} = ?$ (l) $\dfrac{7}{4a} - \dfrac{3}{8a} = ?$

3. Combine the fractions below into a single fraction.

(a) $\dfrac{1}{x} + \dfrac{1}{x+1} = ?$ (b) $\dfrac{2}{x} + \dfrac{1}{x+2} = ?$ (c) $\dfrac{4}{x+1} + \dfrac{3}{x} = ?$

(d) $\dfrac{5}{x} - \dfrac{1}{x+2} = ?$ (e) $\dfrac{5}{x-2} - \dfrac{1}{x} = ?$ (f) $\dfrac{6}{x+3} - \dfrac{4}{3x} = ?$

(g) $\dfrac{6}{x+1} - \dfrac{6}{x} = ?$ (h) $\dfrac{4}{x-5} + \dfrac{2}{x} = ?$ (i) $\dfrac{7}{5x} + \dfrac{4}{x+6} = ?$

(j) $\dfrac{5}{x-7} + \dfrac{7}{2x} = ?$ (k) $\dfrac{6}{x-10} + \dfrac{5}{3x} = ?$ (l) $\dfrac{1}{3x} + \dfrac{2}{x-8} = ?$

4. Simplify each expression below, giving your answer as a single fraction.

(a) $\dfrac{1}{x+1} + \dfrac{1}{x+2} = ?$ (b) $\dfrac{1}{x-1} + \dfrac{1}{x+1} = ?$

(c) $\dfrac{3}{x+2} + \dfrac{4}{x+2} = ?$ (d) $\dfrac{4}{x-2} + \dfrac{2}{x-6} = ?$

(e) $\dfrac{1}{x+3} - \dfrac{2}{x+4} = ?$ (f) $\dfrac{3}{x-7} - \dfrac{2}{x+7} = ?$

(g) $\dfrac{5}{x-4} + \dfrac{3}{8+x} = ?$ (h) $\dfrac{2}{x-4} - \dfrac{4}{x+7} = ?$

(i) $\dfrac{3}{x+6} + \dfrac{5}{x-1} = ?$

(j) $\dfrac{1}{2x+6} + \dfrac{1}{3x-8} = ?$

(k) $\dfrac{1}{2x+5} - \dfrac{1}{5-4x} = ?$

(l) $\dfrac{3}{2x-1} + \dfrac{4}{3x-1} = ?$

(m) $\dfrac{5}{2x+3} + \dfrac{6}{5x-1} = ?$

(n) $\dfrac{6}{3x-7} + \dfrac{2}{2x+3} = ?$

(o) $\dfrac{7}{5x-4} + \dfrac{3}{2x+3} = ?$

5. Simplify each expression.

(a) $\dfrac{x}{x+1} + \dfrac{2x}{x-2} = ?$

(b) $\dfrac{x}{x-7} - \dfrac{2x}{2x+1} = ?$

(c) $\dfrac{x}{x-1} + \dfrac{2x}{3x-1} = ?$

(d) $\dfrac{x}{x+3} + \dfrac{4}{x-1} = ?$

(e) $\dfrac{5x}{x-3} + \dfrac{3x}{x+4} = ?$

(f) $\dfrac{x}{2-x} + \dfrac{4x}{4x-3} = ?$

(g) $\dfrac{2x}{5-x} - \dfrac{3x}{x+1} = ?$

(h) $\dfrac{x}{4+x} - \dfrac{5x}{x+6} = ?$

(i) $\dfrac{x}{x+6} - \dfrac{3}{x-1} = ?$